

Role of doped holes in a U(1) spin liquid

Ki-Seok Kim

Korea Institute for Advanced Study, Seoul 130-012, Korea

(Dated: February 2, 2008)

In the context of the SU(2) slave boson theory we show that condensation of holons can result in the zero mode of a nodal spinon in a single instanton potential. Instanton contribution in the presence of the zero mode induces the 't Hooft effective interaction, here mass to the spinon. We find that the spinon mass is determined by the state of instantons in the presence of the zero mode. The mass corresponds to antiferromagnetic moment of the nodal spinon. Considering the state of instantons, we discuss the possibility of coexistence between antiferromagnetism and d -wave superconductivity in underdoped cuprates.

PACS numbers: 74.20.Mn, 73.43.Nq, 11.10.Kk

High T_c superconductivity (SC) is believed to result from hole doping to an antiferromagnetic Mott insulator ($AFMI$). Hole doping to the $AFMI$ destroys the antiferromagnetic long range order and causes a paramagnetic Mott insulator ($PMMI$) usually dubbed the pseudogap phase. High T_c SC is expected to occur by further hole doping to the $PMMI$ [1]. Recently, the $PMMI$ is proposed to be the U(1) spin liquid ($U1SL$) described by QED_3 in terms of massless Dirac spinons interacting via non-compact U(1) gauge fields, which originates from irrelevance of instanton excitations of compact U(1) gauge fields in the large flavor limit[1, 2]. According to this scenario, high T_c SC arises from hole doping to the $U1SL$.

In the present study we investigate the role of hole doping in the $U1SL$. In the context of the SU(2) slave boson theory[3] doped holes are represented by SU(2) holon doublets. It is the key observation that isospin interactions between spinons and holons can appear in the SU(2) slave boson theory. This new interaction is shown to result in the zero mode of a nodal spinon in a single instanton potential[4], which appears in the SC state resulting from the condensation of holon doublets. In high energy physics the instanton contribution in the presence of the fermion zero mode is well known to induce the 't Hooft effective interaction, here mass to the spinon[5, 6]. We find that the spinon mass is determined by the state of instantons in the presence of the zero mode. The mass corresponds to antiferromagnetic moment of the nodal spinon[7, 8]. Considering the state of instantons, we discuss the possibility of coexistence between antiferromagnetism (AF) and d -wave SC in underdoped cuprates.

We consider an effective Lagrangian describing hole doped $U1SL$ in the context of the SU(2) slave boson

theory[1, 3]

$$Z = \int D\psi_n D z_n D a_\mu e^{-\int d^3x \mathcal{L}},$$

$$\mathcal{L} = \sum_{n,m=1}^2 \left[\bar{\psi}_n \gamma_\mu (\partial_\mu \delta_{nm} + i a_\mu \tau_{nm}^3) \psi_m \right. \\ \left. + |(\partial_\mu \delta_{nm} + i a_\mu \tau_{nm}^3) z_m|^2 + m^2 |z_n|^2 + \frac{u}{2} |z_n|^4 \right. \\ \left. + \frac{1}{2} \sum_{n',m'=1}^2 G \bar{\psi}_n \tau_{nm}^k \psi_m z_{n'}^\dagger \tau_{n'm'}^k z_{m'} \right] + \frac{1}{2e^2} |\partial \times a|^2 (1)$$

Here $\psi_n = \begin{pmatrix} \chi_n^+ \\ \chi_n^- \end{pmatrix}$ is the four component massless Dirac fermion where $n = 1, 2$ represent SU(2) isospin indices. The two component spinors χ_n^\pm are given by $\chi_1^+ = \begin{pmatrix} f_{1e\uparrow} \\ f_{1o\uparrow} \end{pmatrix}$, $\chi_1^- = \begin{pmatrix} f_{2o\uparrow} \\ f_{2e\uparrow} \end{pmatrix}$, $\chi_2^+ = \begin{pmatrix} f_{1e\downarrow}^\dagger \\ f_{1o\downarrow}^\dagger \end{pmatrix}$, and $\chi_2^- = \begin{pmatrix} f_{2o\downarrow}^\dagger \\ f_{2e\downarrow}^\dagger \end{pmatrix}$, respectively. In the spinon field f_{abc} $a = 1, 2$ represent the nodal points of $(\pi/2, \pi/2)$ and $(-\pi/2, \pi/2)$, $b = e, o$, even and odd sites, and $c = \uparrow, \downarrow$, its spin, respectively[7]. The Dirac matrices γ_μ are given by $\gamma_0 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}$, $\gamma_1 = \begin{pmatrix} \sigma_2 & 0 \\ 0 & -\sigma_2 \end{pmatrix}$, and $\gamma_2 = \begin{pmatrix} \sigma_1 & 0 \\ 0 & -\sigma_1 \end{pmatrix}$, respectively, where they satisfy the Clifford algebra $[\gamma_\mu, \gamma_\nu]_+ = 2\delta_{\mu\nu}$ [7]. z_n represents the SU(2) holon doublet with the isospin indices $n = 1, 2$ [3]. m and u denote the mass and self-interaction of the holon, respectively. We model an effective holon potential with easy plane anisotropy resulting from the contribution of high energy fermions[3]. A coupling between the spinon and holon isospins originates from gauge interactions mediated by the time component of SU(2) gauge fields[3]. Similar consideration can be found in Ref. [3]. G denotes the coupling strength between the isospins and τ^k acts on the SU(2) isospin space. As will be seen below, this isospin interaction plays an important role on instanton excitations in the SC state. a_μ is a compact U(1) gauge field in itself. The kinetic energy of the gauge

field arises from particle-hole excitations of high energy quasiparticles[9]. e is an effective internal charge, not a real electric charge.

In passing, we discuss an effective field theory in the SC state of Eq. (1) without the isospin interaction. Holon condensation $\langle z_{1(2)} \rangle \neq 0$ results in the SC state, causing the $U(1)$ gauge field a_μ to be massive via the Anderson-Higgs mechanism. Integration over the massive gauge field gives an effective field theory in terms of electrons $c_n = z_n^\dagger \psi_n$ and holon pairs $z_1 z_2$. The spinons and holons are confined to form internal charge neutral objects. This phase can be considered to be the Higgs-confinement phase in the context of the gauge theory[10]. In the easy plane limit $z_n = e^{i\phi_n}$, the low energy effective Lagrangian is given by

$$\mathcal{L} = \frac{\rho}{2} |\partial_\mu \phi - 2A_\mu|^2 + \frac{i}{2} (\partial_\mu \phi - 2A_\mu) \bar{c}_n \gamma_\mu c_n + \bar{c}_n \gamma_\mu (\partial_\mu + iA_\mu) c_n + \frac{1}{2g} |\bar{c}_1 \gamma_\mu c_1 - \bar{c}_2 \gamma_\mu c_2|^2. \quad (2)$$

Here ϕ is the phase field of the holon pair, $e^{i\phi} = z_1 z_2 = e^{i(\phi_1 + \phi_2)}$. An external electromagnetic field A_μ is introduced. ρ is the stiffness parameter proportional to hole concentration δ and $1/g \sim 1/\rho$, the strength of four fermion interaction. Surprisingly, this effective Lagrangian is nothing but that of the d -wave BCS superconductor[11]. A detailed discussion of this theory can be found in Refs. [11, 12]. In this letter we show that the presence of the isospin interaction can alter this effective field theory completely.

Separating the compact $U(1)$ gauge field a_μ into $a_\mu = a_\mu^{cl} + a_\mu^{qu}$ where a_μ^{cl} represents an instanton configuration and a_μ^{qu} , gaussian fluctuations, and integrating over the massless Dirac spinon field in Eq. (1), we obtain a fermion determinant including the isospin interaction. In order to calculate the determinant we solve an equation of motion in the presence of a single monopole potential $a_\mu^{cl} = a(r) \epsilon_{3\alpha\mu} x_\alpha$ [13] and its corresponding hedgehog configuration $I_\mu^{int} = \frac{1}{2} z_n^\dagger \tau_{nm}^\mu z_m = \Phi(r) x_\mu$ where $a(r)$ and $\Phi(r)$ are proportional to r^{-2} in $r \rightarrow \infty$ with $r = \sqrt{\tau^2 + x^2 + y^2}$ [4, 14]

$$(\gamma_\mu \partial_\mu \delta_{nm} - ia(r)(\gamma \times x)_3 \tau_{nm}^3 + G\Phi(r) x_\mu \tau_{nm}^\mu) \psi_m = E \psi_n. \quad (3)$$

In the absence of the isospin interaction it is well known that there are no fermion zero modes[8]. On the other hand, the presence of the isospin interaction results in a fermion zero mode. In the $SU(2)$ gauge theory of massless Dirac fermions and adjoint Higgs fields interacting via $SU(2)$ gauge fields, Jackiw and Rebbi showed that a Dirac equation coupled to the isospin of the Higgs field has a fermion zero mode in a single magnetic monopole potential[4]. Following Jackiw and Rebbi, we show that Eq. (3) also has a zero mode. We rewrite Eq. (3) in

terms of the two component spinors χ_n^\pm with $E = 0$

$$\begin{aligned} & (\sigma_3 \partial_\tau)_{ij} \chi_{jn}^\pm + (\sigma_2 \partial_x)_{ij} \chi_{jn}^\pm + (\sigma_1 \partial_y)_{ij} \chi_{jn}^\pm \\ & - iay(\sigma_2)_{ij} \chi_{jm}^\pm (\tau^{3T})_{mn} + iax(\sigma_1)_{ij} \chi_{jm}^\pm (\tau^{3T})_{mn} \\ & \pm G\Phi x_\mu \chi_{jm}^\pm (\tau^{\mu T})_{mn} = 0. \end{aligned} \quad (4)$$

Inserting $\chi_{in}^\pm = \mathcal{M}_{im}^\pm \tau_{mn}^2$ with a two-by-two matrix \mathcal{M}^\pm into the above, we obtain

$$\begin{aligned} & \sigma_3 \partial_\tau \mathcal{M}^\pm + \sigma_2 \partial_x \mathcal{M}^\pm + \sigma_1 \partial_y \mathcal{M}^\pm \\ & + iay \sigma_2 \mathcal{M}^\pm \sigma^3 - iax \sigma_1 \mathcal{M}^\pm \sigma^3 \mp G\Phi \mathcal{M}^\pm x_\mu \sigma^\mu = 0 \end{aligned} \quad (5)$$

Now the isospin matrices and the Dirac matrices are indistinguishable[4]. Finally, representing the matrix \mathcal{M}^\pm in $\mathcal{M}_{im}^\pm = g^\pm \delta_{im} + g_\mu^\pm \sigma_{im}^\mu$, we obtain coupled equations of motion for the numbers g^\pm and g_μ^\pm

$$\begin{aligned} & (\partial_\tau \mp G\Phi \tau) g^\pm - i(\partial_x + ax \pm G\Phi y) g_1^\pm \\ & + i(\partial_y + ay \pm G\Phi x) g_2^\pm = 0, \\ & (\partial_x - ax \mp G\Phi y) g^\pm + i(\partial_\tau \pm G\Phi \tau) g_1^\pm \\ & - i(\partial_y - ay \pm G\Phi x) g_3^\pm = 0, \\ & (\partial_y - ay \mp G\Phi x) g^\pm - i(\partial_\tau \pm G\Phi \tau) g_2^\pm \\ & + i(\partial_x - ax \pm G\Phi y) g_3^\pm = 0, \\ & (\partial_\tau \mp G\Phi \tau) g_3^\pm + (\partial_x + ax \mp G\Phi y) g_2^\pm \\ & + (\partial_y + ay \mp G\Phi x) g_1^\pm = 0. \end{aligned} \quad (6)$$

These equations yield the following zero mode equations

$$\begin{aligned} & (\partial_\tau + G\Phi \tau) g^- = 0, \\ & (\partial_x - ax + G\Phi y) g^- = 0, \\ & (\partial_y - ay + G\Phi x) g^- = 0. \end{aligned} \quad (7)$$

The zero mode solution g^- is given by $g^- \sim \exp \left[- \int d\tau G\Phi(r) \tau + \int dx (a(r)x - G\Phi(r)y) + \int dy (a(r)y - G\Phi(r)x) \right]$. Without the isospin interaction it can be easily seen that there exist no normalizable fermion zero modes[8]. The existence of the zero mode makes the fermion determinant zero in the single monopole excitation. As a result the condensation of magnetic monopoles is forbidden. It is well known that the monopole condensation causes confinement of charged particles[2, 15]. The suppression of monopole condensation results in deconfinement[16] of internally charged particles, here the spinons and holons. This deconfined SC state completely differs from the usual one corresponding to the Higgs-confinement phase described by Eq. (2). Below we discuss an effective field theory to describe this unusual SC state.

In high energy physics it is well known that the instanton contribution in the presence of the fermion zero mode gives rise to an effective interaction to the fermions[5, 6]. This interaction is usually called the 't Hooft effective interaction. In order to obtain the effective fermion interaction it is necessary to average the partition function in

Eq. (1) over various instanton and anti-instanton configurations. Following Ref. [6], first we consider a partition function in a single instanton potential

$$\begin{aligned} Z_\psi &= \int D\psi_n e^{-\int d^3x \bar{\psi}_n \gamma_\mu \partial_\mu \psi_n \left(m - V^I[\psi_n] \right)}, \\ V^I[\psi_n] &= \int d^3x \left(\bar{\psi}_n(x) \gamma_\mu \partial_\mu \Phi_n^I(x) \right) \\ &\times \int d^3y \left(\bar{\Phi}_n^I(y) \gamma_\mu \partial_\mu \psi_n(y) \right). \end{aligned} \quad (8)$$

Here Φ_n^I is the zero mode obtained from Eq. (7). A fermion mass m is introduced. Later the chiral limit $m \rightarrow 0$ will be chosen. The effective action including the effective potential $V^I[\psi_n]$ in Eq. (8) gives a correct green function in a single instanton potential[6], $S^I(x, y) = \langle \psi_n(x) \bar{\psi}_n(y) \rangle = -\frac{\Phi_n^I(x) \bar{\Phi}_n^I(y)}{m} + S_0(x, y)$ with the bare propagator $S_0(x, y) = (\gamma_\mu \partial_\mu)^{-1} \delta(x - y)$. Thus the partition function Eq. (8) can be used for instanton average[6]. The partition function in the presence of N_+ instantons and N_- anti-instantons can be easily built up[6]

$$\begin{aligned} Z_\psi &= \int D\psi_n D a_\mu^{qu} e^{-\int d^3x \bar{\psi}_n \gamma_\mu (\partial_\mu - i a_\mu^{qu}) \psi_n} \\ &\times \left(m - \langle V^I[\psi_n] \rangle \right)^{N_+} \left(m - \langle V^I[\psi_n] \rangle \right)^{N_-}. \end{aligned} \quad (9)$$

Here we admit a non-compact U(1) gauge field a_μ^{qu} representing gaussian fluctuations. Below the index qu is omitted. $\langle \dots \rangle$ means averaging over the individual instantons. Introducing instanton averaged non-local fermion vertices $Y_\pm = -V \langle V^I[\psi_n] \rangle = -\int d^3z z_{I(\bar{I})} V^{I(\bar{I})}[\psi_n]$ with volume V , where $z_{I(\bar{I})}$ represent instanton center positions[6], we obtain a partition function in the chiral limit $m \rightarrow 0$

$$\begin{aligned} Z_\psi &= \int D\psi_n D a_\mu e^{-\int d^3x \bar{\psi}_n \gamma_\mu (\partial_\mu - i a_\mu) \psi_n} \\ &\times \int \frac{d\lambda_\pm}{2\pi} \int d\Gamma_\pm e^{i\lambda_+(Y_+ - \Gamma_+) + N_+ \ln \frac{\Gamma_+}{V} + (+ \rightarrow -)}. \end{aligned} \quad (10)$$

Integration over λ_\pm and Γ_\pm recovers Eq. (9) in the chiral limit. In the thermodynamic limit $N_\pm, V \rightarrow \infty$ and N_\pm/V fixed, integration over Γ_\pm and λ_\pm can be performed by the saddle point method[6]. Integrating over Γ_\pm first, we obtain

$$\begin{aligned} Z_\psi &= \int \frac{d\lambda_\pm}{2\pi} e^{N_+ \left(\ln \frac{N_+}{i\lambda_+ V} - 1 \right) + (+ \rightarrow -)} \\ &\times \int D\psi_n D a_\mu e^{-\int d^3x \bar{\psi}_n \gamma_\mu (\partial_\mu - i a_\mu) \psi_n + i\lambda_+ Y_+ + i\lambda_- Y_-} \end{aligned} \quad (11)$$

An explicit calculation for the instanton average shows that the vertex Y^\pm corresponds to a mass[6], $Y^\pm = \int \frac{d^3k}{(2\pi)^3} (2\pi\rho F(k))^2 \bar{\psi}_n \frac{1 \pm \gamma_5}{2} \psi_n$ with $\gamma_5 = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$ [7]. Here $F(k)$ is associated with the fermion zero mode in

the effective potential $V^I[\psi_n]$ in Eq. (8). In the present paper we do not perform an explicit calculation for the instanton average in Y_\pm and thus we do not know an exact form of $F(k)$. Our objective is to see how the 't Hooft interaction appears as an instanton effect. Here ρ is the size of an instanton. Owing to the neutrality condition of magnetic charges $N_+ = N_- = N/2$ is obtained in Eq. (11), where N is the total number of instantons and anti-instantons. The saddle point solution for $\lambda_+ = \lambda_- \equiv \lambda$ in Eq. (11) gives rise to the cancellation of the γ_5 term in the mass, causing the momentum dependent mass $m(k) = m_\psi F^2(k)$ with $m_\psi = \lambda(2\pi\rho)^2$ [6]. The mass m_ψ is determined by the saddle point equation for λ usually called the self-consistent gap equation[6]

$$\frac{8}{N/V} \int \frac{d^3k}{(2\pi)^3} \frac{m^2(k)}{k^2 + m^2(k)} = 1. \quad (12)$$

Ignoring the momentum dependence by setting $F(k) = 1$ for simplicity, we obtain the mass $m_\psi = \frac{\pi}{2\Lambda^{1/2}} \left(\frac{N}{V} \right)^{1/2}$ with the momentum cut-off Λ in small mass limit. Since the mean density of instantons is proportional to the instanton fugacity, $N/V \sim y_m = e^{-S_{inst}}$ with an instanton action $S_{inst} \sim 1/e^2$ [15, 17], the fermion mass is roughly given by $m_\psi \sim y_m^{1/2}$. We obtain the effective Lagrangian in terms of the Dirac spinon ψ_n with the 't Hooft effective mass m_ψ interacting via the non-compact U(1) gauge field a_μ , $\mathcal{L}_\psi = \sum_{n,m=1}^2 \left[\bar{\psi}_n \gamma_\mu (\partial_\mu \delta_{nm} + i a_\mu \tau_{nm}^3) \psi_m + m_\psi \bar{\psi}_n \psi_n \right]$. Despite the mass term we cannot say that the spinon is really massive. We should show that the instanton fugacity y_m is non-zero. y_m would be determined self-consistently in the presence of holon contributions.

Including the holon contribution, the non-linear σ model with the easy plane anisotropy and performing a standard duality transformation[17, 18, 19], we obtain the total effective Lagrangian in terms of the Dirac spinons and holon vortices with the electromagnetic field A_μ [20]

$$\begin{aligned} \mathcal{L} &= \sum_{n=1}^2 \left[|(\partial_\mu - i c_{n\mu}) \Phi_n|^2 + m_\Phi^2 |\Phi_n|^2 + \frac{u_\Phi}{2} |\Phi_n|^4 \right. \\ &\quad \left. - i(\partial \times c_n)_\mu A_\mu + \frac{1}{2\rho} |\partial \times c_n|^2 \right] \\ &\quad + \frac{1}{2e^2} |\partial \times a|^2 - i(\partial \times a)_\mu (c_{1\mu} - c_{2\mu}) \\ &\quad + \sum_{n,m=1}^2 \left[\bar{\psi}_n \gamma_\mu (\partial_\mu \delta_{nm} + i a_\mu \tau_{nm}^3) \psi_m + m_\psi \bar{\psi}_n \psi_n \right] \end{aligned} \quad (13)$$

Here $\Phi_{1(2)}$ is the vortex field with isospin \uparrow (\downarrow) (isospin \uparrow (\downarrow) meron field[18]) and $c_{1(2)\mu}$, its vortex gauge field mediating interactions between the vortices. m_Φ and u_Φ are the mass and self-interaction of the vortices, respectively. $\rho \sim \delta$ is the coupling strength between the vortex and vortex gauge field. The presence of the fermion zero mode in a single instanton potential is the key ingredient

resulting in Eq. (13). In the absence of the fermion zero mode the term $-y_m(\Phi_1\Phi_2^\dagger + \Phi_1^\dagger\Phi_2)$ is usually generated in the dilute approximation of instantons[17, 18, 19]. A renormalization group (*RG*) study shows instanton condensation ($y_m \rightarrow \infty$)[19] inducing vortex pair condensation $\langle \Phi_1\Phi_2^\dagger \rangle \neq 0$ in the *SC* state[17, 19]. As a result the Higgs-confinement phase arises[17, 19]. This state is described by the holon pairs z_1z_2 [Eq. (2)] in the Higgs field representation. On the other hand, in the presence of the fermion zero mode this term makes the partition function zero and thus does not contribute to our effective Lagrangian Eq. (13). In the dilute approximation of instantons the fugacity y_m appears only in the fermion mass. At this level of approximation it is difficult to determine the instanton fugacity, i.e., the state of instantons. As a matter of fact it is a long standing unsolved problem to determine the state of instantons in the presence of matter fields. Generally speaking, there are two possible instanton states resulting in deconfined *SC*; one is a dipolar phase ($y_m \rightarrow 0$) and the other, a "liquid" phase ($0 < y_m < \infty$). The latter does not appear in the Abelian Higgs model[19] (without fermions). But, in the present model we do not have any evidence to exclude this instanton state. In (2+1) dimensions the basic trend is confinement, i.e., $y_m \rightarrow \infty$ [10, 17] away from quantum criticality[2, 18, 19]. Owing to the confinement tendency we should consider dense instantons. But, the presence of the fermion zero mode does not allow instanton condensation. In the dense limit a new phase is expected instead of plasma and dipolar phases. There exist some reports about a new phase in the two dimensional Coulomb gas when the density of particles is high[21]. Furthermore, a new fixed point with non-zero instanton fugacity was recently reported even in the *QED*₃ with only massless Dirac fermions[22]. In this letter we consider a liquid phase of instantons, i.e., $y_m \neq 0$. We view the emergence of a liquid state as the proximate effect of the Higgs-confinement phase in the presence of the fermion zero mode. In the dipolar phase the spinon mass vanishes because of $m_\psi \sim y_m^{1/2} \rightarrow 0$. The resulting effective field theory is completely the same as Eq. (13) except zero spinon mass. We do not exclude the possibility of this dipolar phase. We will discuss this plausible state in a separate publication, including phase transitions between the three different *SC* phases.

The mass corresponds to the antiferromagnetic moment of the nodal fermion[7, 8]. If instantons are in a liquid state, the *AF* of the nodal fermions can coexist with the *d-wave SC* in underdoped cuprates[23]. The mass can be considered as an evidence of deconfinement in the underdoped *SC* phase. Thus, if the *AF* is observed in the *SC* phase, deconfinement of the spinons and holons is expected to occur. Many recent experiments have reported the coexistence of the *AF* and *SC*[24]. Our new

SC may have a chance to be applicable.

Since the Dirac fermions are massive in the present consideration, they can be safely integrated out. As a result the Maxwell kinetic energy $\mathcal{L}_a = \frac{1}{2\tilde{e}^2}|\partial \times a|^2$ with $\tilde{e}^2 = 12\pi m_\psi$ [7] is generated. Integrating over the internal gauge field a_μ , we obtain a mass term for the vortex gauge fields, $\frac{e_{eff}^2}{2}|c_{1\mu} - c_{2\mu}|^2$ with an effective internal charge $e_{eff}^2 = (\tilde{e}^2\tilde{e}^2)/(e^2 + \tilde{e}^2)$. The mass is a relevant parameter in the *RG* sense, thus admitting us to set $c_{1\mu} = c_{2\mu} \equiv c_\mu$ in the low energy limit. An effective Lagrangian is obtained to be in the *SC* state

$$\mathcal{L}_{SC} = \sum_{n=1}^2 \left[|(\partial_\mu - ic_\mu)\Phi_n|^2 + m_\Phi^2|\Phi_n|^2 + \frac{u_\Phi}{2}|\Phi_n|^4 \right] - i2A_\mu(\partial \times c)_\mu + \frac{1}{\rho}|\partial \times c|^2. \quad (14)$$

In the coupling $-i2A_\mu(\partial \times c)_\mu$ an electric charge 2 originates from both the z_1 and z_2 bosons. $2e_{el}$ electric charge infers that a vortex quantum is $hc/2e_{el}$. Although the underdoped *SC* state is argued to be the deconfinement phase, the vortex quantum is not hc/e_{el} but $hc/2e_{el}$. This is nothing but the meron-type vortex discussed in Ref. [25]. The superconductor to insulator transition induced by the meron vortices is expected to fall into the XY universality class[12, 23]. The above holon vortex Lagrangian is just dual to the non-linear σ model with a non-compact U(1) gauge field in Eq. (1). This Lagrangian was recently studied by the present author using a *RG* analysis[19]. In the study the author showed that the quantum critical point is governed by the XY fixed point. This result seems to be consistent with experiments for YBCO[26].

We would like to comment that the present SU(2) formulation is valid only in underdoped region[3]. The effect of SU(2) symmetry breaking may be studied by introducing the Zeeman terms, $-H_\psi\bar{\psi}\tau^3\psi$ and $-H_z z^\dagger\tau^3 z$, where $H_{\psi(z)}$ is an effective "magnetic field" proportional to hole concentration. They would be important at large doping. $-H_\psi\bar{\psi}\tau^3\psi$ is expected to make the fermion zero mode disappear. This can be checked by investigating the equation of motion [Eq. (3)] in the presence of the Zeeman term. The role of $-H_z z^\dagger\tau^3 z$ is not clear at present. The disappearance of the fermion zero mode will cause the Higgs-confinement phase described by Eq. (2). We anticipate a quantum phase transition between the deconfined *SC* [Eq. (13)] and the *BCS* one [Eq. (2)] at some critical doping inside the *SC* dome. This interesting possibility will be studied near future.

K.-S. Kim especially thanks Dr. Yee, Ho-Ung for helpful discussions of the fermion zero mode and 't Hooft interaction.

- [3] P. A. Lee et al., Phys. Rev. B **57**, 6003 (1998); P. A. Lee and N. Nagaosa, Phys. Rev. B **68**, 024516 (2003).
- [4] R. Jackiw and C. Rebbi, Phys. Rev. D **13**, 3398 (1976).
- [5] G. 't Hooft, Phys. Rev. D **14**, 3432 (1976).
- [6] D. Diakonov, Lectures at the Enrico Fermi School in Physics, Varenna, (1995); hep-ph/9602375.
- [7] D. H. Kim and P. A. Lee, Annals Phys. **272**, 130 (1999).
- [8] J. B. Marston, Phys. Rev. Lett. **64**, 1166 (1990).
- [9] The kinetic energy is given by $-\frac{1}{e^2} \sum_n \cos(\partial \times a)_n$ in lattice, where n is its dual lattice. We approximate this to the Maxwell form in the continuum limit, keeping the compactness of the gauge field.
- [10] E. Fradkin and S. H. Shenker, Phys. Rev. D **19**, 3682 (1979).
- [11] D. H. Lee, Phys. Rev. Lett **84**, 2694 (2000).
- [12] K.-S. Kim et al., Phys. Rev. B **69**, 014504 (2004).
- [13] An instanton solution representing a tunnelling event between energetically degenerate but topologically inequivalent gauge vacua is nothing but a magnetic monopole in two space dimensions and one imaginary time dimension[15].
- [14] L. H. Ryder, Quantum Field Theory (2nd., Cambridge University Press, 1996).
- [15] A. M. Polyakov, Gauge Fields and Strings (ch.4), harwood academic publishers. (1987).
- [16] Even in the deconfinement phase the Coulomb interaction is logarithmically confining in two space dimensions.
- [17] N. Nagaosa and P. A. Lee, Phys. Rev. B **61**, 9166 (2000).
- [18] T. Senthil et al., Science **303**, 1490 (2004); T. Senthil et al., Phys. Rev. B **70**, 144407 (2004).
- [19] K.-S. Kim, cond-mat/0406511.
- [20] The isospin interaction is expected to be marginally irrelevant in the RG sense. Thus it does not affect the SC transition. It can be ignored in the absence of the single instanton excitations.
- [21] J.-R. Lee and S. Teitel, Phys. Rev. B **46**, 3247 (1992); P. Gupta and S. Teitel, Phys. Rev. B **55**, 2756 (1997); The new phase is the ionic lattice state. Although it is not a liquid phase owing to different Coulomb potentials, we have a clue to expect another new phase.
- [22] F. S. Nogueira and H. Kleinert, cond-mat/0501022.
- [23] K.-S. Kim et al., cond-mat/0404527.
- [24] J. E. Sonier et al., Science **292**, 1692 (2001); Y. Sidis et al., Phys. Rev. Lett. **86**, 4100 (2001); H. A. Mook et al., Phys. Rev. B **64**, 012502 (2001); M. -H. Julien et al., Phys. Rev. B **63**, 144508 (2001); S. Ono et al., Phys. Rev. Lett. **85**, 638 (2000); Ch. Niedermayer et al., Phys. Rev. Lett. **80**, 3843 (1998).
- [25] P. A. Lee and X.-G. Wen, Phys. Rev. B **63**, 224517 (2001).
- [26] F. S. Nogueira, Phys. Rev. B **62**, 14559 (2000); references therein; D. J. Lee and I. D. Lawrie, Phys. Rev. B **64**, 184506 (2001); references therein.